ON THE THIN CELL METHOD IN TIME DOMAIN SPECTROSCOPY

M.J.C. VAN GEMERT and W.H. DE JEU
Philips Research Laboratories, Eindhoven, The Netherlands

Received 17 July 1974

A simple physical picture is given of Fellner-Feldegg's thin cell method in time domain spectroscopy. From this picture an accurate analytical relation is derived for the total reflection coefficient.

1. Introduction

As an alternative to frequency domain methods time domain spectroscopy (TDS) methods can be applied to the study of the dielectric behaviour of polar materials [1–3]. These methods all involve the propagation of a step voltage in a coaxial line partly filled with the dielectric material. From the change in the shape of the step after, e.g., reflection against the interface air–dielectric, the dielectric properties of the material can be found.

In this letter a simple physical picture of Fellner-Feldegg's thin cell method [4, 5] will be given in terms of lumped elements. From this picture an expression for the total reflection coefficient will be derived. This new relation will be compared with the exact behaviour and with Fellner-Feldegg's thin cell relation.

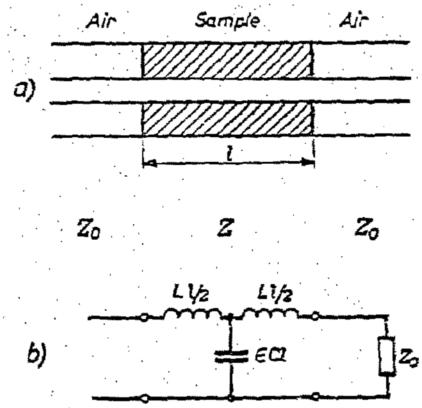

Fig. 1. (a) Coaxial line containing dielectric sample. (b) Equivalent circuit using lumped elements.

2. Lumped element picture of the thin cell method

In the thin cell method the sample is placed as indicated in fig. 1a. Taking the sample length to be much smaller than the wavelengths of interest the equivalent circuit of fig. 1*b* can be expected to apply. L and C are given by [6]

$$L = (\mu/2\pi)\ln(b/a)\,, \tag{1a}$$

$$C = 2\pi\varepsilon/\ln(b/a)\,. \tag{1b}$$

ε and μ are the permittivity and permeability of vacuum, b and a the outer and inner diameter of the coaxial line. Z_0, the characteristic impedance of the empty line, is given by

$$Z_0 = (L/C)^{1/2} = (\mu c/2\pi)\ln(b/a)\,, \tag{2}$$

where c is the velocity of light in vacuum. The reflection coefficient of the above network is

$$R(\mathrm{i}\omega) = [Z(\mathrm{i}\omega) - Z_0]/[Z(\mathrm{i}\omega) + Z_0]\,, \tag{3}$$

with

$$Z(\mathrm{i}\omega) = \mathrm{i}\omega Ll/2 + Z_0/(1 + \mathrm{i}\omega\varepsilon ClZ_0)\,. \tag{4}$$

Using eqs. (1), (2) and (4), eq. (3) leads to

$$R(\mathrm{i}\omega) = \frac{\mathrm{i}\omega l}{2c}\left[1 - \epsilon + \left(\frac{\mathrm{i}\omega l}{2c}\right)^2 \epsilon\right] \times \left[1 + \frac{\mathrm{i}\omega l}{2c}(1+\epsilon) + \left(\frac{\mathrm{i}\omega l}{2c}\right)^2 \left(2 + \frac{\mathrm{i}\omega l}{2c}\right)\epsilon\right]^{-1} . \quad (5)$$

This equation has the interesting property that the relation between $R(\mathrm{i}\omega)$ and $\epsilon(\mathrm{i}\omega)$ is *bi-linear*, which is important for practical applications. For small values of $\omega l/2c$, eq. (5) can be approximated by

$$R(\mathrm{i}\omega) = \frac{\mathrm{i}\omega l}{2c}\,\frac{1-\epsilon}{1+(\mathrm{i}\omega l/2c)(1+\epsilon)} . \quad (6)$$

Relations (5) and (6) have been compared with the exact reflection coefficient given by [4, 5]

$$R(\mathrm{i}\omega) = \rho(\mathrm{i}\omega)\,\frac{1-\exp\left[-(2l/c)\mathrm{i}\omega\epsilon^{1/2}\right]}{1-\rho^2(\mathrm{i}\omega)\exp\left[-(2l/c)\mathrm{i}\omega\epsilon^{1/2}\right]} , \quad (7)$$

where

$$\rho(\mathrm{i}\omega) = (1-\epsilon^{1/2})/(1+\epsilon^{1/2}) , \quad (8)$$

and with Fellner-Feldegg's original thin cell relation

$$R(\mathrm{i}\omega) = \mathrm{i}\omega(l/2c)\,(1-\epsilon) . \quad (9)$$

This comparison has been made by computing numerically the step response $r(t)$, using for $R(s)$ the relations (5)–(7) and (9), respectively. The step response is defined as

$$r(t) = \frac{1}{2\pi\mathrm{i}}\int_{d-\mathrm{i}\infty}^{d+\mathrm{i}\infty} \mathrm{d}s\,\exp(st)\,[R(s)/s] \equiv L^{-1}[R(s)/s] , \quad (10)$$

where $s = \gamma + \mathrm{i}\omega$ (γ, ω real) and L^{-1} denotes the inverse Laplace operator. For convenience we have chosen a Debye permittivity relation to represent $\epsilon(\mathrm{i}\omega)$, thus

$$\epsilon(\mathrm{i}\omega) = \epsilon_\infty + (\epsilon_0 - \epsilon_\infty)/(1+\mathrm{i}\omega\tau_0) , \quad (11)$$

where ϵ_∞, ϵ_0 and τ_0 are the permittivity at high frequencies, the static permittivity and the relaxation time, respectively. The numerical computations have been carried out with a procedure developed by Stehfest [7]. The results, using $\epsilon_0 = 20$, $\epsilon_\infty = 4$ and $2l/c\tau_0 = 0.1$, are shown in fig. 2. We conclude that for this particular case where the thin cell relation eq. (9) gives already a substantial deviation from the exact response, eqs. (5) and (6) are still very good approximations. Unfortunately these equations cannot be used in general to analyse the permittivity directly in the time domain.

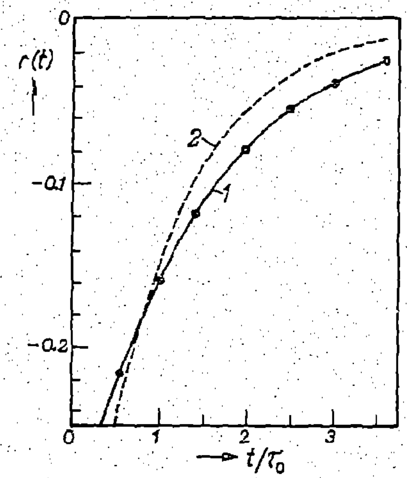

Fig. 2. Calculated step response behaviour for $\epsilon_0 = 20$, $\epsilon_\infty = 4$, $2l/c\tau_0 = 0.1$. 1: Exact response using eq. (7). 2: Response using the original thin cell relation eq. (9). •: Response using eqs. (5) and (6). For this case no significant difference is observed between these two relations.

The good results of eqs. (5) or (6) can be understood from the fact that their Taylor expansions in $\omega l/2c$ are up to second order equal to the Taylor expansion of the exact reflection coefficient eq. (7). As an alternative, therefore, this expansion up to second order in $\omega l/2c$ could be also used. However, the relation between $R(\mathrm{i}\omega)$ and $\epsilon(\mathrm{i}\omega)$ is then quadratic, while conductivity cannot be included as well as in eqs. (5) or (6) (see the next section).

3. Conductive materials

If the low frequency conductivity σ cannot be neglected relative to the dielectric loss, the term $\sigma/\epsilon\mathrm{i}\omega$ has to be added to $\epsilon(\mathrm{i}\omega)$ in all the relations used. Thus eq. (9) changes to [4]

$$R(\mathrm{i}\omega) = \mathrm{i}\omega\{(l/2c)[1-\epsilon(\mathrm{i}\omega)] - (l/2c)\,\sigma/\epsilon\mathrm{i}\omega\} , \quad (12)$$

leading to [4, 5]

$$r(t) = (l/2c)\,L^{-1}[\epsilon_\infty - \epsilon(s)] - (l/2c)\,\sigma/\epsilon . \quad (13)$$

Hence the influence of the conductivity is to produce an offset of the base line. Therefore, if the thin cell approximation is sufficiently accurate the pulse response of the permittivity can be determined separately from the value of the conductivity. However, the exact relation describing the offset of the base line is [5]

$$\lim_{t\to\infty} r(t) = \lim_{s\to 0} R(s) = -\frac{(l/2c)\sigma/\varepsilon}{1+(l/2c)\sigma/\varepsilon}. \quad (14)$$

Introducing conductivity in eq. (6) gives

$$R(i\omega) = \frac{i\omega l}{2c}\left[\frac{1-\epsilon-\sigma/\varepsilon i\omega}{1+(i\omega l/2c)(1+\epsilon)+(l/2c)\sigma/\varepsilon}\right]. \quad (15)$$

From this equation the offset of the base line is predicted to be equal to the exact relation (14). The same holds true if conductivity is introduced in eq. (5). Consequently the lumped element model (fig. 1b) predicts correctly the value of the offset of the base line, independent of the values of l and σ. Furthermore, a numerical calculation of $r(t)$ computed from eq. (15) gives the same sort of agreement with the exact step response as shown in fig. 2 for the case without conductivity.

We conclude that the lumped element picture of fig. 1b provides simple and accurate formulae to replace the more approximate original thin cell relation. The practical applicability of the formulae obtained will be discussed elsewhere [8].

Acknowledgement

The authors thank T.A.C.M. Claasen and T.E. Rozzi for valuable discussions on transmission line theory.

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